JRF Math (Algebra)
Nov 18,2016
Instructor: N.S.N. Sastry

Note: Answer *all* questions. Your answers should be *complete, clear* and *to the point*.

1. Define the presentation of a group. write the set of all pairwise distinct elements of the group given by the
presentation $\langle x. y |\ x^2. y^2 \rangle$. (10 marks)

2. Show that the group generated by any two involutions of a group is dihedral. Determine the set of its
involutions. (10 marks)

3. Define a primitive permutation group. Show that anormal subgroup of a finite group acting primitively on a finite set X acts either trivially or transitivelyon X.(10 marks)

4. Define primitive ring and a simple ring. Give an example of a primitive ring which is not a simple ring. Is the converse true? Justify. (10 marks) — 3+4+3

5. Construct a Sylow $p-$subgroup of S_{p^2}, p a prime. Justify. (20 marks)

6. Let G be the group of upper triangular nonsingular $n \times n$- matrices with entries from a finite field $\mathbb{F}_{p^t}$, p a prime, with ones on the diogonal.Show that the G-module given by the natural action of G on $\mathbb{F}_{p^t}^n$ is has no decomposable submodules. (20 marks)

7.Determine all nonabelian groups of order 12. Justify your answer. (20 marks)

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